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Lesson 22: Modeling an Invasive Species Population

Student Outcomes

* Students apply knowledge of exponential functions and transformations of functions to a contextual situation.

Lesson Notes

The lionfish is an invasive species that began appearing in the western Atlantic Ocean in 1985 and now have large populations throughout the Caribbean as well as along eastern coastline of the United States. They have recently been spotted as far north as New York and Rhode Island. More information can be found at the site <http://nas.er.usgs.gov/queries/FactSheet.aspx?speciesID=963>.

*Scaffolding:*

When grouping students, consider their reading skill. Create groups ahead of time with varying ability levels so that they can assist students who may struggle with the reading comprehension aspect of this exercise.

To maximize class time, create groups ahead of time and place students in groups as they come in the classroom.

The focus of this lesson is on MP.4, modeling with mathematics. Additionally, students will need to employ MP.2 as they move back and forth between a real world context and an algebraic model.

Classwork

Opening (4 minutes)

Ask students to read through the introductory paragraph independently, or read it aloud as they read silently. After students have had a chance to read through the introductory paragraph, show them a picture of a lionfish and the interactive map at the site <http://nas.er.usgs.gov/queries/SpeciesAnimatedMap.aspx?speciesID=963>.

Discussion (3 minutes)

Motivate the lesson by asking students to discuss the problems inherent with introducing a non-native invasive species to a region. Why should we care about these problems?

* The lionfish has no natural predator in this region to help control its population.
* Lionfish reproduce quickly and eat large amounts of the fish around the reef or area they inhabit.

Mathematical Modeling Exercise (25 minutes)

Have students work in groups. Circulate throughout the room asking students about the problem. Make sure students are providing thorough written explanations when called for. To engage students with MP.1, set an expectation and encourage students to persist in struggling with the problem for some time without teacher assistance. If you feel students are equipped for further challenge, consider simply presenting the data and asking them to develop a model, without the scaffolding questions provided. If graphing calculators or spreadsheet software are not available, have students use points from their graph to write the equation.

* Why did you decide the data was linear/exponential?
* After seeing the scatterplot, did you change your mind?
* What do you think the graph will look like in the future? Will it keep going up? Level off? Why?

Mathematical Modeling Exercise

The lionfish is a fish that is native to the western Pacific Ocean. The lionfish began appearing in the western Atlantic Ocean in 1985. This is probably because people bought them as pets and then dumped them in waterways leading to the ocean. Because it has no natural predators in this area, the number of lionfish grew very quickly and now has large populations throughout the Caribbean as well as along the eastern coastline of the United States and the Gulf of Mexico. They have recently been spotted as far north as New York and Rhode Island.

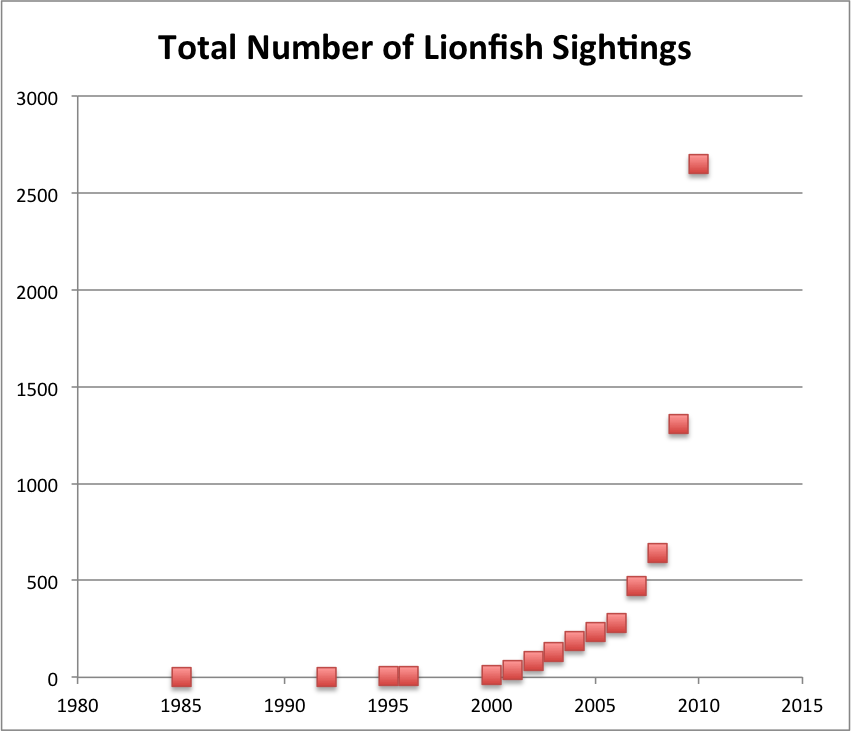
The table below shows the number of new sightings by year reported to NAS (Nonindigenous Aquatic Species), which is a branch of the U.S. Geological Survey Department.

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| --- | --- | --- |
| Year | Number of New Sightings | Total Number of Sightings |
| 1985 |  |  |
| 1992 |  |  |
| 1995 |  |  |
| 1996 |  |  |
| 2000 |  |  |
| 2001 |  |  |
| 2002 |  |  |
| 2003 |  |  |
| 2004 |  |  |
| 2005 |  |  |
| 2006 |  |  |
| 2007 |  |  |
| 2008 |  |  |
| 2009 |  |  |
| 2010 |  |  |

1. Complete the table by recording the total number of sightings for each year.
2. Examine the total number of sightings data. Which model appears to be a better fit for the data, linear or exponential? Explain your reasoning.

Exponential. For example, for the last three years the total number of sightings appears to have doubled each year.

1. Make a scatterplot of the year versus the total number of sightings.

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1. Based on the scatterplot, either revise your answer from Exercise 2 or explain how the scatterplot supports your answer from Exercise 2.

Answers will vary.

1. On the scatterplot, draw a smooth curve that best fits the data.

The smooth curve should look similar to the above scatterplot.

1. From your table, calculate the average rate of change in total number of sightings for each of the following time intervals.
   1. 1995–2000 b. 2000–2005 c. 2005–2010

1. How do the average rates of change help to support your argument of whether a linear or exponential model is better suited for the data?

If the model total number of sightings was growing linearly, then the average rate of change would be constant. Instead, it appears to be growing multiplicatively, indicating an exponential model.

*Scaffolding:*

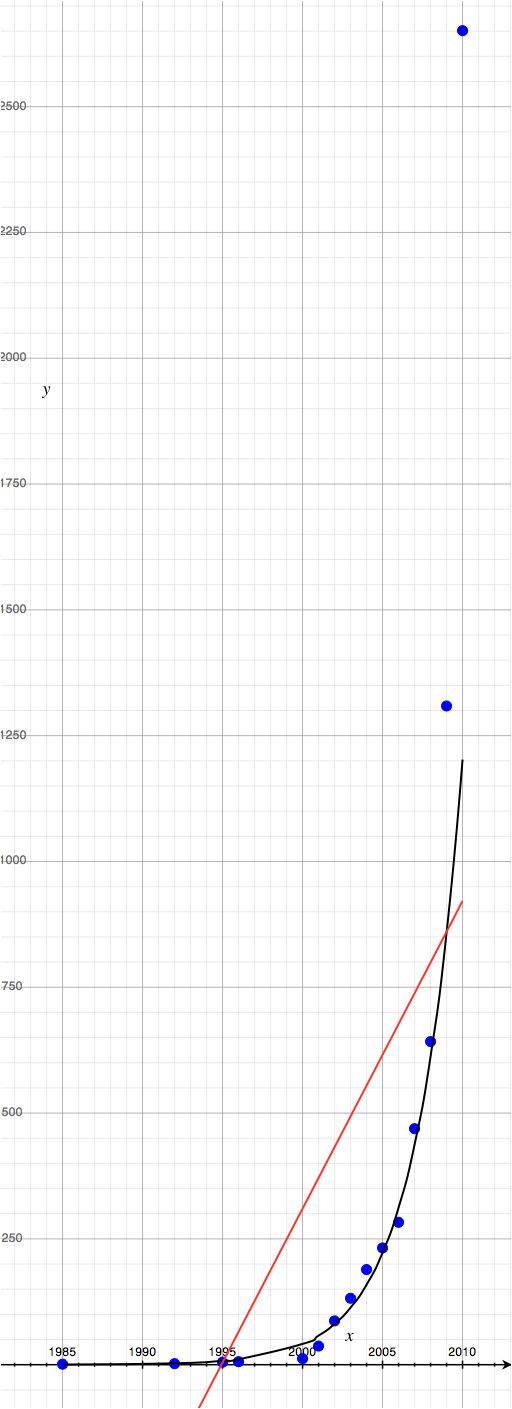
This lesson could be extended to a student project. Early finishers could work on researching other non-native invasive species found in the U.S.

1. Use the regression feature of a graphing calculator to find an equation that models the number of lionfish sightings each year.

Answers will vary depending on the technology used. If the selected technology is not sensitive enough to display the equation for the exponential model, provide students with the equation below.

Exponential Model: Linear Model:

Bivariate Data Points

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1. Use your model to predict the total number of lionfish sightings by the end of 2013.

Exponential: total sightings.   
Linear: total sightings.

1. The actual number of sightings as of July 2013 was . Does it seem that your model produced an accurate prediction? Explain.

Answers will vary. See closing below.

Closing (8 minutes)

After students have completed the problem, discuss the following:

* Which model would be a better fit for the data – linear or exponential? Why?
  + *Exponential. The rate of change was not constant, which would make it linear. The average rates of change got progressively larger. The graph had the basic shape of an exponential growth curve.*
* What do you think the graph will look like in the future? Will it keep going up? Level off? Why?
  + *Answers will vary.*
* Was your model accurate in predicting the number of sightings for 2013?
  + *If students used the graphing calculator to find the equation, the prediction is pretty close (3,147–3,307) but does underestimate the actual number especially considering that the number is only for the first half of 2013. If students did not use a graphing calculator, their answers will vary. Compare several equations.*
* Could your model accurately predict the future number of sightings?
  + *Answers will vary.*
* What are the possible implications if the number continues to increase exponentially?
  + *The population will become so large that it will threaten other marine life especially around coral reefs.*

Exit Ticket (5 minutes)

Name \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Lesson 22: Modeling an Invasive Species Population

Exit Ticket

1. For the equation found in Exercise 8, explain the parameters of the equation within the context of the problem.
2. Given each of the following, describe what features in the data or graph make it apparent that an exponential model would be more suitable than a linear model?
   1. The table of data.
   2. The scatterplot.
   3. The average rates of change found in question 6.
3. Use your equation from Exercise 8 to predict the number of lionfish sightings by year 2020. Is this prediction accurate? Explain.

Exit Ticket Sample Solutions

1. For the equation found in Exercise 8, explain the parameters of the equation within the context of the problem.

, at , there were virtually no lionfish in the Atlantic Ocean.

, the population of lionfish grew by this constant quotient of change every year.

1. Given each of the following, describe what features in the data or graph make it apparent that an exponential model would be more suitable than a linear model?
   1. The table of data.

The values of the lionfish sightings grow by a quotient of change rather than a constant rate of change.

* 1. The scatterplot.

The shape of the scatterplot looks more like an exponential curve than a line.

* 1. The average rates of change found in question 6.

The three sets of average rates of change would have been the same, or close to the same, had the situation been more suited to a linear model.

1. Use your equation from question 8 to predict the number of lionfish sightings by year 2020. Is this prediction accurate? Explain.

; this answer assumes that the conditions that allowed for a constant quotient of change remain the same until 2020. This is probably not a realistic scenario, change in ocean temperature, food availability and other needs for continued exponential growth would have remain the same to yield this projected population.

Problem Set Sample Solutions

Another invasive species problem: kudzu

Kudzu, a perennial vine native to Southeast Asia, now covers a large area of the southern United States. Kudzu was promoted as a forage crop and an ornamental plant when it was introduced to the U.S. at the Philadelphia Centennial Exposition in 1876. Many southern farmers were encouraged to plant kudzu for erosion control from the mid 1930’s to the mid 1950’s. In 1953, kudzu was removed from the U.S. Department of Agriculture’s list of permissible cover plants due to its recognition as an invasive species.

1. Look up information about kudzu in the United States on Wikipedia and write a short (1–2 pg.) report on the growth of kudzu since its introduction. In your report, choose a function (linear or exponential) to model and graph the growth of kudzu (in hectares) in the U.S. per year over the past half-century or so. Remember to cite your sources!

To the teacher: Point out that the report is not about getting a correct answer, but how they explain and manipulate the data that they find. Hence, it is important for them to cite their sources. (Note: a rough estimate of how kudzu spreads, 50,000 hectares/year, is on the Wiki page—challenge them to find out more than that.)